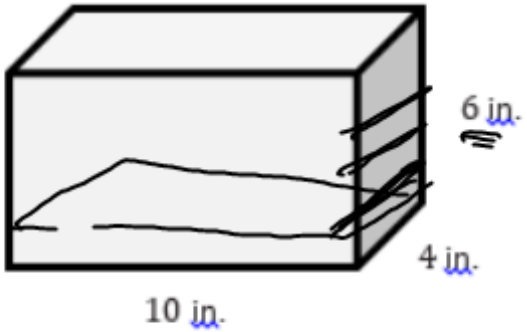
Lesson 11: Volume with Fractional Edge Lengths and Unit Cubes

Classwork

Opening Exercise

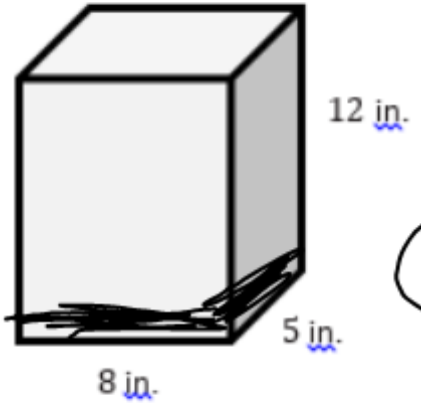
Which prism will hold more 1 in. × 1 in. × 1 in. cubes?

12
40
480



240 cubes

40 cubes on bottom



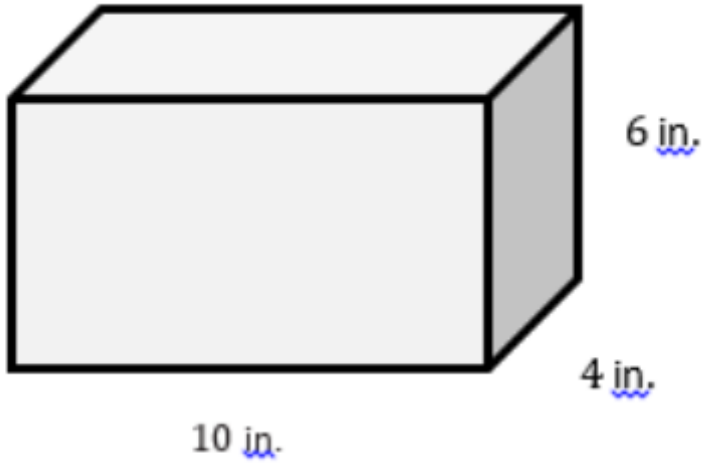
480 cubes

40 cubes

How many more cubes will the prism hold?

Example 1

A box with the same dimensions as the prism in the Opening Exercise will be used to ship miniature dice whose side lengths have been cut in half.



Example 1

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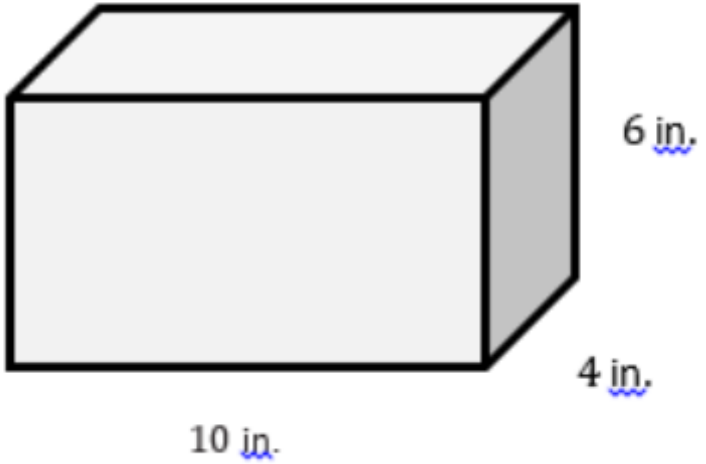
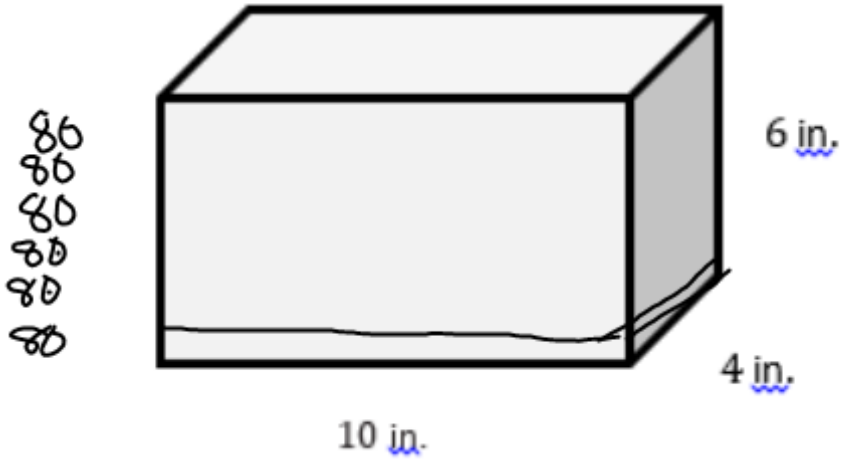


The dice are $\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. cubes. How many dice of this size can fit in the box?

Example 1

A box with the same dimensions as the prism in the Opening Exercise will be used to ship miniature dice whose side lengths have been cut in half. The dice are $\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. cubes. How many dice of this size can fit in the box?

480 cubes

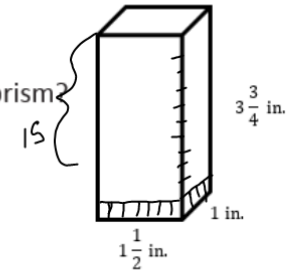


A $\frac{1}{4}$ in. cube was used to pack the prism.

How many $\frac{1}{4}$ in. cubes will it take to fill the prism?

What is the volume of the prism?

How is the number of cubes related to the volume?



$6 \cdot 4 \cdot 15$

$6 \cdot 60$

360 cubes

$4 \cdot \frac{1}{4} = 1$

$2 \cdot \frac{1}{4} = \frac{1}{2}$

Volume of 1 cube $= \frac{1}{4}\text{in} \cdot \frac{1}{4}\text{in} \cdot \frac{1}{4}\text{in} = \frac{1}{64}\text{in}^3$

$360 \cdot \frac{1}{64}\text{in}^3 = \frac{360}{64}\text{in}^3 = 5\frac{40}{64}\text{in}^3 = 5\frac{5}{8}\text{in}^3$

Volume of 1 cube

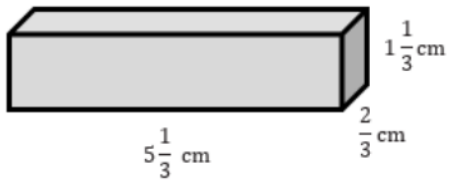
$V = L \cdot W \cdot H$

$V = \left(1\frac{1}{2}\text{ in} \times 1\text{ in} \times 3\frac{3}{4}\text{ in}\right)$

$V = \frac{3}{2}\text{in} \cdot \frac{1}{1}\text{in} \cdot \frac{15}{4}\text{in}$

$= \frac{45}{8}\text{in}^3 = 5\frac{5}{8}\text{in}^3$

1. Use the prism to answer the following questions.



   a. Calculate the volume.

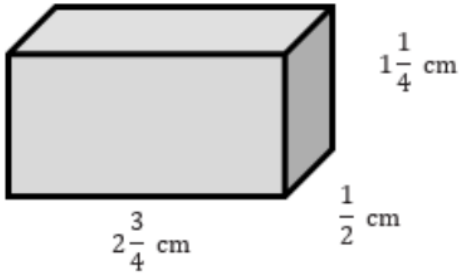
   b. If you have to fill the prism with cubes whose side lengths are less than 1 cm, what size would be best?

   c. How many of the cubes would fit in the prism?

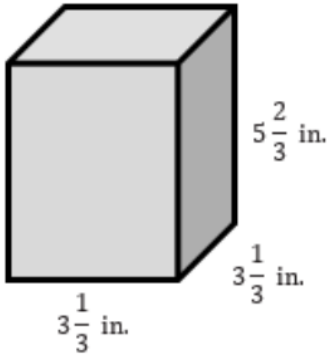
   d. Use the relationship between the number of cubes and the volume to prove that your volume calculation is correct.

Calculate the volume of the following rectangular prisms.

a.



b.



Classwork

Opening Exercise

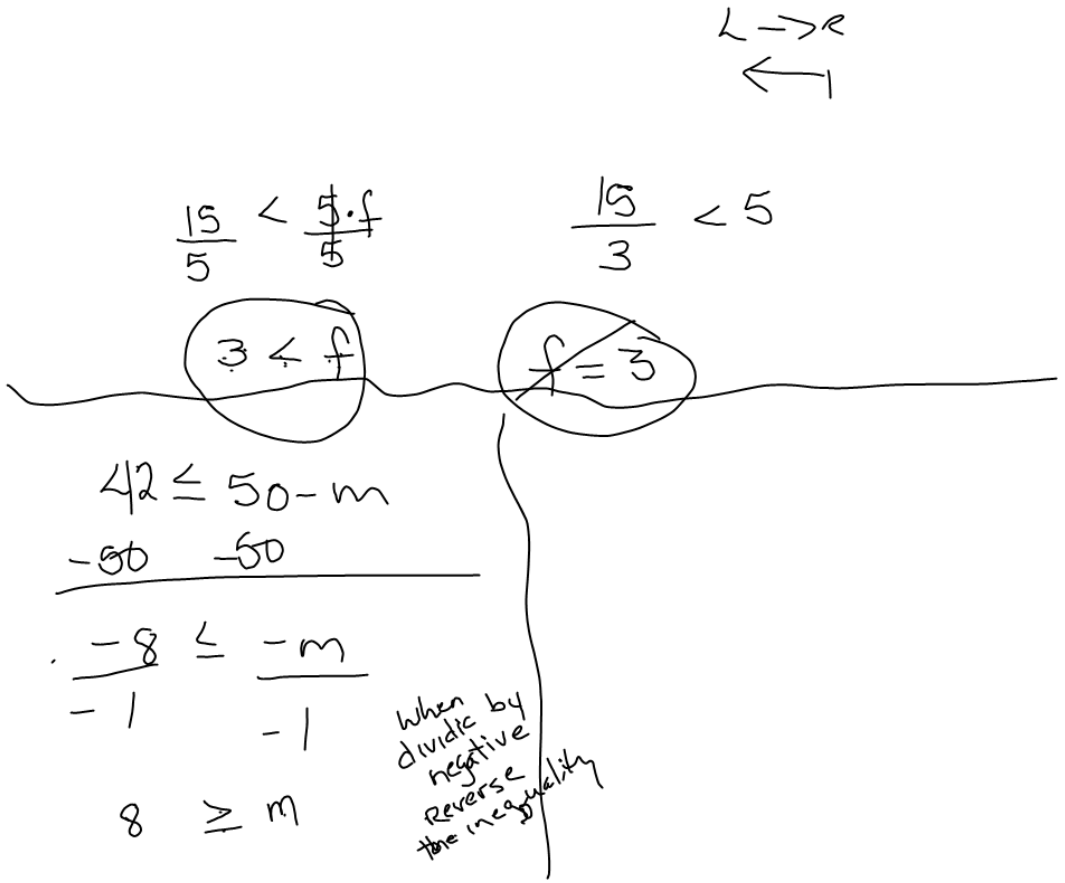
Identify a value for the variable that would make each equation or inequality into a true number sentence. Is this the only possible answer? State when the equation or inequality is true using equality and inequality symbols.

a. $3 + g = 15$

b. $30 > 2d$ $\quad d < 15$

c. $\frac{15}{f} < 5$ $\quad f > 3$

d. $42 \leq 50 - m$



Example 1

Each of the following numbers, if substituted for the variable, makes one of the equations below into a true number sentence. Match the number to that equation: $3, 6, 15, 16, 44$.

a. $n + 26 = 32$

b. $n - 12 = 32$

c. $17n = 51$

d. $4^2 = n$

e. $\frac{n}{3} = 5$

Lesson Summary

VARIABLE: A *variable* is a symbol (such as a letter) that represents a number (i.e., it is a placeholder for a number).

A variable is a placeholder for "a number" that does not "vary."

EXPRESSION: An *expression* is a numerical expression or a result of replacing some (or all) of the numbers in a numerical expression with variables.

EQUATION: An *equation* is a statement of equality between two expressions.

If A and B are two expressions in the variable x, then $A = B$ is an equation in the variable x.

Find the solution to each equation.

1. $4^3 = y$
2. $8a = 24$
3. $32 = g - 4$
4. $56 = j + 29$
5. $\frac{48}{r} = 12$
6. $k = 15 - 9$
7. $x \cdot \frac{1}{5} = 60$
8. $m + 3.45 = 12.8$
9. $a = 1^5$

1) $-6k + 7k$

2) $12r - 8 - 12$

3) $n - 10 + 9n - 3$

4) $-4x - 10x$

5) $-r - 10r$

6) $-2x + 11 + 6x$

7) $11r - 12r$

8) $-v + 12v$

1) $-6k + 7k$

2) $12r - 8 - 12$

3) $n - 10 + 9n - 3$

4) $-4x - 10x$

5) $-r - 10r$

6) $-2x + 11 + 6x$

7) $11r - 12r$

8) $-v + 12v$

P
E
M
D
A
S

P
E
M • D
A • S

1) $18 \div 3 \times 2 + 4 - 2$

$6 \times 2 + 4 - 2$

2) $4[(15 - 3^2) + 8(2)]$

3) $\dfrac{5^2 + 15}{15 \bullet 2 - 3}$

4) $2^2 + 8 \times 3 + 2 - 4$

$2^2 + 8 \times 3 + 2 - 4$

5) $16 \div 2 \times 4 - 6 + 1$

6) $\dfrac{2^3 + 2}{4 - 1}$

1) $18 \div 3 \times 2 + 4 - 2$

$6 \times 2 + 4 - 2$

$12 + 4 - 2$

$16 - 2$

14

2) $4[(15 - 3^2) + 8(2)]$

$4[(15 - 9) + 8(2)]$

$4[(6) + 16]$

$4(22)$

88

$$\frac{5^2+15}{15\bullet 2-3} =$$

$$\frac{25+15}{30-3} = \frac{40}{27}$$

24

27

28

$\frac{41}{27}$

$\frac{40}{27}$

$1\frac{13}{27} = \frac{40}{27}$

4) $2^2 + 8 \times 3 + 2|-4|$

$2^2 = 2 \cdot 2$

$3^2 = 3 \cdot 3$

$2^2 + 8 \cdot 3 + 2 - 4$

$4 + 8 \cdot 3 + 2 - 4$

$4 + 24 + 2(-4)$

26

5) $16 \div 2 \times 4 - 6 + 1$

$8 \times 4 - 6 + 1$

$32 - 6 + 1$

$26 + 1$

27

COMBINING LIKE TERMS ------ WORKSHEET #1

Directions: Simplify using either algebra tiles or shapes.

1. $5d + 3d =$ _____

2. $6x + 5y - 2x =$ _____

3. $n + 3n - 4m + 6m =$ _____

1. $5d + 3d =$ _____

2. $6x + 5y - 2x =$ _____

3. $n + 3n - 4m + 6m =$ _____

4. $2p + 6 - 7p + 10 =$ _____

5. $f - 6 - 8f + 6 =$ _____